Efficient Body of Revolution Finite-Difference Time-Domain Modeling of Integrated Lens Antennas

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***Abstract*—An efficient body of revolution finite-difference time-domain (BOR-FDTD) method for the analysis of radiation properties of integrated lens antennas is presented in this paper. By neglecting most of the reactive power of the planar feed and by expanding the filtered source currents into azimuthal modes, lenses with both rotationally and nonrotationally symmetric planar feeds can be handled. It appears that three to four azimuthal modes are sufficient to adequately model the magnetic currents of a double-slot feed. Therefore, compared to a full three–dimensional (3-D) numerical method, the implementation of the proposed method is very time and memory efficient. If only the radiation properties are required, the model described here can also be applied efficiently to other axially symmetric geometries with an asymmetric feeding structure.**

***Index Terms*—BOR-FDTD, Fourier transform, integrated lens antennas.**

I. Introduction

INTEGRATED lens antennas are a very attractive solution for applications in the (sub)millimeter-wave region. Usually, the analysis of these antennas is performed by a hybrid method-of-moments (MoM) and geometrical optics/physical optics (GO/PO) technique [1], [2]. The justification for using GO inside the dielectric lens is that the lens usually is large in terms of the dielectric wavelength and therefore the lens surface is located in the far field of the planar feed. For the analysis of relatively small integrated lens antennas, the GO/PO method was modified into a more accurate PO/PO method [3].

However, the applicability of both methods is limited by the fact that they can only handle simple homogeneous dielectric lenses. Another complex issue is the modeling of (multiple) internal reflections and of matching layers for relatively small lenses. To overcome these difficulties, a full-wave method like finite element method (FEM) [4], MoM or finite-difference time-domain (FDTD) can be used. The full three–dimensional (3-D) modeling of these antennas usually is very memory intensive ($D/\lambda > 10$ and $\varepsilon_r > 10$). Therefore, in this paper, an alternative full-wave method, the body of revolution FDTD (BOR-FDTD), is proposed that permits the analysis of general rotationally symmetric integrated lens antennas with circularly symmetric (e.g., annular slot and circular patch) feeds. To handle lens antennas with nonrotationally symmetric planar feeds (e.g., double slot, double dipole or square patch), a Fourier expansion of the filtered source currents is applied. As opposed to techniques where the original source is used, this results in fewer azimuthal modes to be taken into account.

The proposed model can relatively easy handle lenses with varying dielectric constants (flat lenses), integrated lens antennas combined with objective lenses [5] and multiple internal reflections [1].

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II. Proposed BOR-FDTD Model

Expressing the electric and magnetic fields in cylindrical coordinates gives the following Fourier series [6]:

$$\vec{E}(\rho, \varphi, z) = \sum_{m=0}^{\infty} \vec{E}_m^e(\rho, z)\cos m\varphi + \vec{E}_m^o(\rho, z)\sin m\varphi \quad (1)$$

$$\vec{H}(\rho, \varphi, z) = \sum_{m=0}^{\infty} \vec{H}_m^e(\rho, z)\cos m\varphi + \vec{H}_m^o(\rho, z)\sin m\varphi \quad (2)$$

where m is the azimuthal mode index. The even and odd modes are denoted by superscripts e and o, respectively. After substitution of (1) and (2) into Maxwell's equations, the BOR-FDTD update equations are found [6]. A perfectly matched layer boundary is implemented to limit the computational domain. Furthermore, curved dielectric and metallic structures are treated according to the conformal method described in [7].

To apply (1) and (2) to the modeling of integrated lens antennas, a nonrotationally symmetric feed has to be replaced by its equivalent azimuthal mode content. However, if the planar feed structure consists of rapid spatial variations, the number of relevant azimuthal modes is very high. Since, in this paper, only the radiation properties of these integrated lens antennas are of interest, the reactive power of the planar feed can be neglected. By neglecting this contribution, the number of modes to be taken into account significantly reduces.

In the following, a magnetic current source is considered, but electric currents can be treated in a similar way. The Fourier transform of the original magnetic current distribution is written as

$$\vec{\mathbf{M}}(k_\rho, \varphi') = \int_0^{2\pi}\int_0^{\infty} \vec{M}(\rho, \varphi)e^{jk_\rho \rho \cos(\varphi' - \varphi)}\rho d\rho d\varphi. \quad (3)$$

Then, after applying a lowpass filter with a cut-off frequency (k_{co}) larger than $k_d = 2\pi\sqrt{\varepsilon_r}/\lambda_0$ and inverse Fourier transform, the resulting equation for the band-limited magnetic current is

$$\vec{M}_F(\rho, \varphi) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{\infty} \vec{\mathbf{M}}(k_\rho, \varphi')\mathbf{H}(k_\rho) \times e^{-jk_\rho \rho\cos(\varphi' - \varphi)} k_\rho\, dk_\rho\, d\varphi' \quad (4)$$

with $\mathbf{H}(k_\rho)$ being the transfer function of the lowpass filter. The filter characteristic is defined in the spectral domain as

$$\mathbf{H}(k_\rho) = \begin{cases} 1 & k_\rho \le k_{co} \\ \cos^2\left[\frac{\pi(k_\rho - k_{co})}{2\Delta k_{ro}}\right], & k_{co} < k_\rho < k_{co} + \Delta k_{ro}. \end{cases} \quad (5)$$

The resulting (filtered) magnetic current distribution is then expanded into its azimuthal modes, according to (1), and the radial mode distributions are used as magnetic source in the BOR-FDTD update equations for each separate mode m. Finally, adding the contributions of each mode gives the total radiation pattern.

III. Results for an Annular-Slot Feed

In order to verify the validity of the method described in II, the far-field patterns of an integrated lens antenna with an annular slot feed are computed. The reason for choosing this planar antenna is two-fold: the current distribution of the source resembles a delta function (in ρ) that can be used to build any source distribution according to

$$\vec{M}(\rho, \varphi) = \sum_{m=0}^{\infty}\left[\int_0^{\infty}\delta(\rho - \rho_0)\vec{M}_m^e(\rho)\, d\rho\right]\cos m\varphi. \quad (6)$$

If it can be proven that the proposed method works for different mode numbers and for different values of ρ_0, then the method will also work for general source distributions (e.g., a double slot). A second reason for selecting the annular slot is that the true physical source (without approximations) can be implemented directly in the BOR-FDTD algorithm, giving a reference far-field pattern.

To reduce the complexity of the analysis, it is assumed that the magnetic current in the annular slot can be approximated by a single azimuthal mode

$$M_\varphi(\rho, \varphi) = \delta(\rho - \rho_0)\cos m\varphi \quad (7)$$

with ρ_0 the (mean) radius of the annular slot. It should be noted that the existence of higher-order azimuthal modes is not a limitation of the proposed method. By means of (3), the Fourier transform of the magnetic source can be computed analytically. Applying (4) together with the azimuthal mode expansion gives the required input current sources for the BOR-FDTD algorithm.

An analysis of sufficiently large annular-slot fed lens antennas (seven wavelengths in diameter) showed for arbitrary m and ρ_0 an exact match between the beam patterns obtained with filtered source currents and those obtained with the original source currents. The results were independent of the width of the cosine roll-off (Δk_{ro}) of the filter. A gradual decrease of the lens size showed a dependence of the selected values for k_{co} and Δk_{ro}. The reason for this is that the filtered source current distribution extends over a much larger area in the spatial domain than the original source. If the filtered source extends further than the edge of the lens, information of the source will be lost and as a consequence the computed far-field patterns will differ.

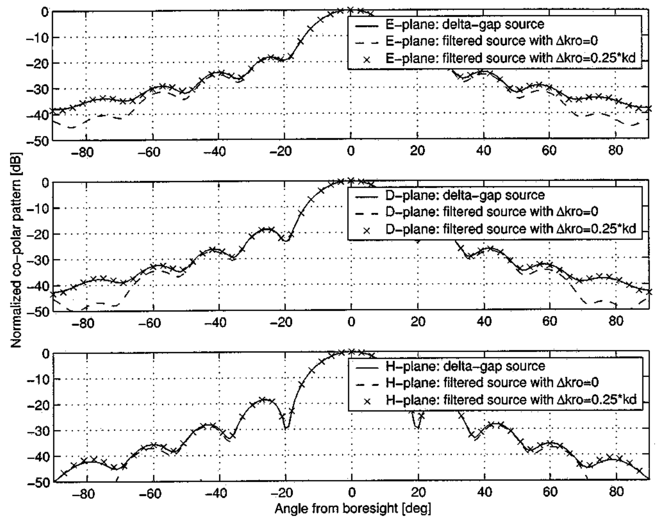

Fig. 1. Normalized far-field patterns of a 2.0-mm-diameter silicon ($\varepsilon_r = 11.7$) elliptical lens antenna at 500 GHz, $m = 1$.

To limit the spatial size of the filtered currents, both the cut-off frequency and the cosine roll-off width can be changed. As an example, the effect of the lowpass filter roll-off width Δk_{ro} is shown in Fig. 1, where $k_{co} = k_d$. In this figure, the diagonal plane (45°-plane), between the E- and H-plane, is denoted by D-plane. For the comparison, an elliptical silicon ($\varepsilon_r = 11.7$) lens antenna is selected with a diameter of 2.0 mm and a frequency of operation of 500 GHz. To minimize reflection losses at the lens-air boundary, a quarter-wave matching layer is included.

In Fig. 1, it can be seen that by increasing the width of the filter roll-off from 0 to $0.25k_d$, the predicted far-field patterns for the filtered source better agree with the delta source. Most likely, the smooth transition from the radiative region to the reactive region is the most determining factor for a small spatial source region. Of course, the lowpass filter design is not restricted to this kind of roll-off shapes, because what matters is that all the radiative power is taken into account and that the filter provides a smooth transition.

IV. Results for a Double-Slot Feed

In the previous section, the proposed method is verified for a circular symmetric feed. To show the validity of the method for nonrotationally symmetric feeds, the convergence of the far-field patterns of a double-slot fed lens antenna is tested by shifting the cut-off (spatial) frequency of the lowpass filter. The diameter of the elliptical shaped silicon lens is 2.0 mm while the double slot has a length of 168 micron and a separation (width) of 93 micron.

In Fig. 2, the resulting co-polar patterns are depicted for a filter cut-off frequency of $0.8k_d$, $0.9k_d$, $1.0k_d$ and $1.1k_d$. Because the design of this double slot nearly gives a rotationally symmetric pattern in the dielectric, only the azimuthal modes $m = 1$, 3, and 5 have to be taken into account. As expected, the patterns converge for a cut-off frequency beyond $1.0k_d$. As an estimate of the efficiency of the proposed method, a reduction factor of more than 50 in computer memory and time is achieved compared to a 3-D FDTD algorithm.

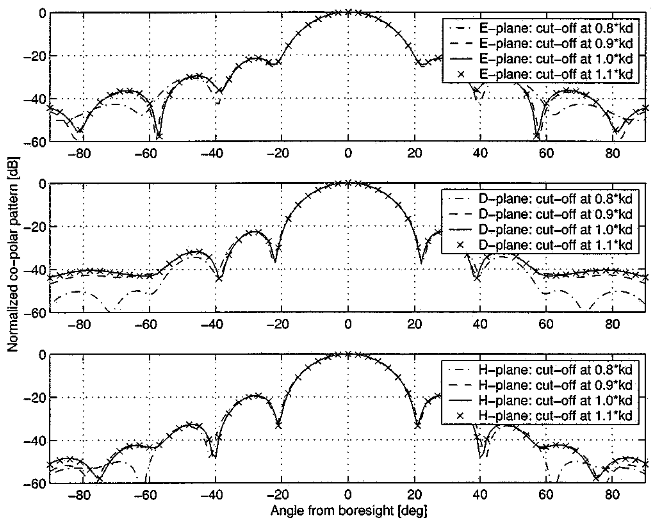

Fig. 2. Normalized far-field patterns of a 2.0-mm-diameter silicon ($\varepsilon_r = 11.7$) elliptical lens antenna at 500 GHz, $m = 1$, 3, and 5.

V. Conclusion

To effectively analyze integrated lens antennas with and without circular symmetric planar feeds, the BOR-FDTD method can be applied together with an azimuthal mode expansion of the feed source. To limit the number of azimuthal modes for a nonrotationally symmetric feed (e.g., double slot), the planar feed currents have to be filtered in the spectral domain. The cut-off frequency of the filter should be higher than the wavenumber in the dielectric to include all of the radiative power. For a particular double-slot design, the number of relevant modes appeared to be limited to three, which makes the implementation of the BOR-FDTD algorithm very time and memory efficient compared to a full 3-D numerical method. A memory- and time-saving factor of 50–100 can easily be found if the diameter of the lens is in the order of 5–10 wavelengths.

To minimize the truncation effect of relatively small lens antennas (diameter of several wavelengths), a smooth transition from the radiative to the reactive region of the source currents is necessary. This function can be provided by a lowpass filter and will limit the size of the filtered source currents in the spatial domain.

References

[1] M. J. M. van der Vorst, P. J. I. de Maagt, and M. H. A. J. Herben, “Effect of internal reflections on the radiation properties and input admittance of integrated lens antennas,” *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 1696–1704, Sept. 1999.

[2] M. J. M. van der Vorst, P. J. I. de Maagt, A. Neto, A. L. Reynolds, R. M. Heeres, W. Luinge, and M. H. A. J. Herben, “Effect of internal reflections on the radiation properties and input impedance of integrated lens antennas-Comparison between theory and measurements,” *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 1118–1125, June 2001.

[3] J. Bray and L. Roy, “Analysis of electrically small slot-fed substrate lens antennas using the physical optics hybrid method,” *Electromagnetics*, vol. 20, pp. 243–256, 2000.

[4] A. D. Greenwood and J.-M. Jin, “Finite-element analysis of complex axisymmetric radiating structures,” *IEEE Trans. Antennas Propagat.*, vol. 47, pp. 1260–1266, Aug. 1999.

[5] M. J. M. van der Vorst, P. J. I. de Maagt, and M. H. A. J. Herben, “Electromagnetic modeling of objective lenses in combination with integrated lens antennas,” in *9th Int. Symp. Space Tera-hertz Tech.*, Pasadena, CA, 1998, pp. 389–395.

[6] A. Taflov and S. C. Hagness, *Computational Electrodynamics: The Finite Difference Time Domain Method*. Norwood, MA: Artech House, 2000, ch. 12.

[7] W. Yu, D. Arakaki, and R. Mittra, “On the solution of a class of large body problems with full or partial circular symmetry by using the finite-difference time-domain (FDTD) method,” *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 1810–1817, Dec. 2000.